NETWORK TECHNOLOGIES

SHANNON'S LIMITS APPLIED TO CABLE NETWORKS ABOVE 1 GHZ

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EXECUTIVE SUMMARY

Because of improved Cable taps and improved amplifiers, it is becoming feasible to use the bandwidth over 1 GHz for communications services. The number one application for higher channel capacity is high-speed Internet service. Unfortunately, elevated cable attenuation and combined noise work together, along with limited transmit power to reduce data throughput, for both upstream and downstream signals. This discussion looks at the Shannon-Hartley Theorem for the case where a user's upstream transmit power is limited, but excess bandwidth is available (i.e., 1-1.7 GHz).

1 INTRODUCTION

Node sizes are becoming smaller and amplifier cascades are shortening. Operators are interested in both a peak offered data capacity to one customer and total capacity available to a group of subscribers. Analysis of plants, such as illustrated in Figure 1, shows that in many cases with noise-funneling and high attenuation, the maximum data capacity for many subscribers is going to be determined by available transmit power, which may be assumed to be in the range of 65 dBmV. A similar logic follows for downstream operation.

So, a new challenge is to find the maximum data capacity when transmit power is limited but there is additional available bandwidth. This is generally not a problem with DOCSIS services, where a reduction in peak transmit power during ranging is normal and there is no additional bandwidth available.

2 DISCUSSION

Using sparse parity code FEC (forward error correction) methods, communications are now able to approach within a few tenths of a dB of the Shannon limit for a channel.

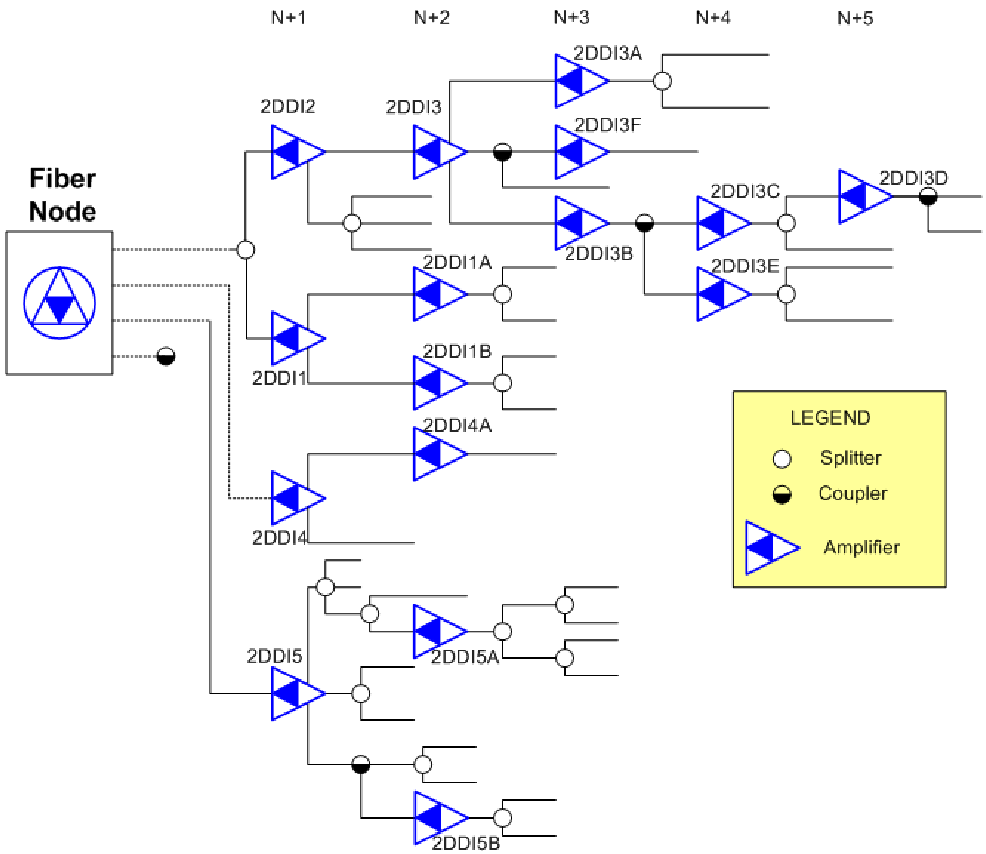

Figure 1. A Typical Node That Can Be Used for Analysis

Shannon's well-known theorem is:

$$W = B \cdot log_2(1 + \frac{S}{N})$$

Where W is the data capacity of a channel in bits per second, below which data can be sent without error. B is the bandwidth of the signal in Hertz, S is the power of the signal and N is the power of the noise in the occupied band.

If the power spectral density of the noise is constant across the frequency range of interest, the noise power (N) can then be expressed as the product of the channel bandwidth (B) and the noise power spectral density (d).

$$N = d \cdot B$$

Substituting into Shannon's theorem:

$$W = B \cdot log_2(1 + \frac{S}{d \cdot B})$$

If the transmit power (S) is held constant, then the expression can be simplified as:

$$W = B \cdot log_2(1 + \frac{k}{B})$$

with:

$$k = \frac{S}{d}$$

When this expression is plotted over a bandwidths ranging from 1 to 32 MHz the results are somewhat surprising. See Figure 2.

For this plot, an assumed value for B is 6 MHz with a signal to noise ratio of 30dB, so we can solve for k:

$$k = \frac{S}{d} = \frac{d_s B_s}{d} = B_s \cdot SNR$$

$$k = 6 \cdot 10^6 \cdot 1000 = 6 \cdot 10^9$$

Plotting, we find:

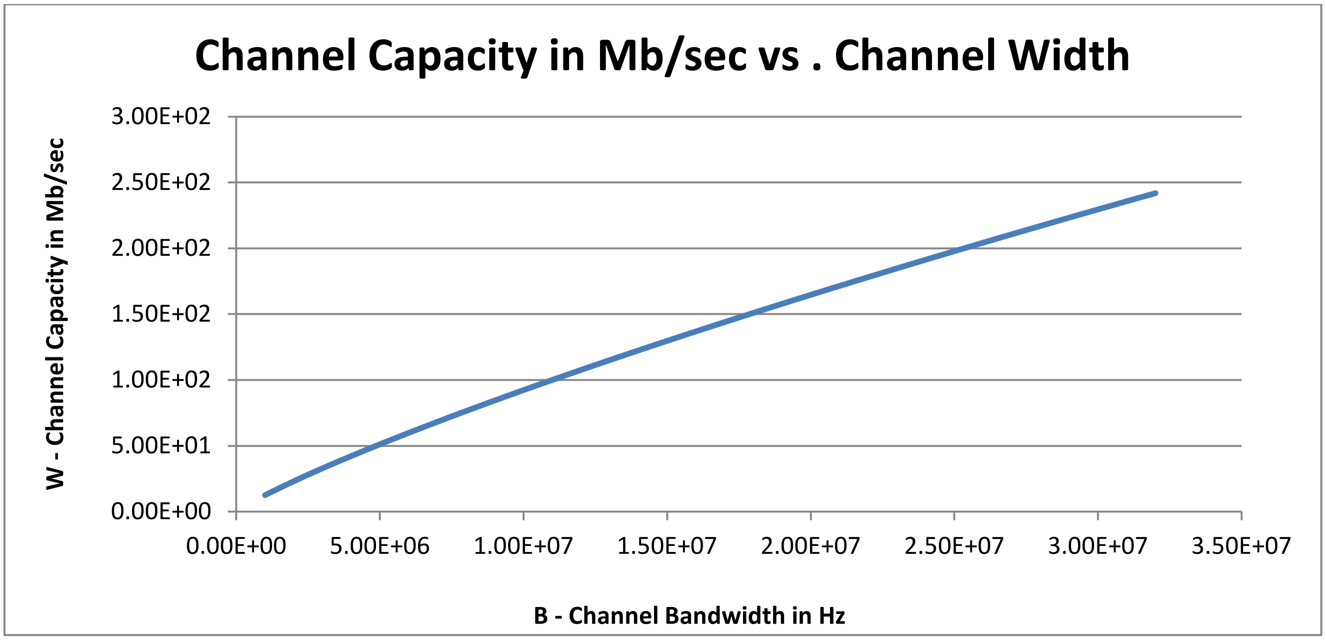

Figure 2. Shannon's Capacity Limit for a Channel with Fixed Transmit Power but Variable Bandwidth.

This plot shows that channel capacity monotonically increases as the channel bandwidth increases from 1 to 32 MHz. This demonstrates that, when excess bandwidth is available, constraining a signal to a narrow band reduces capacity severely. For reference on this graph, at 6 MHz bandwidth the Shannon capacity is 59.8 Mb/sec. This compares to a raw 256-QAM capacity of about 39 Mb/sec.

Another example to contemplate is what would happen in a DOCSIS channel if transmit power increased by 6dB. Signal to noise ratio could go from 22 to 28dB.

This would allow a 256-QAM carrier to be used instead of a 64-QAM carrier. This improvement would result in a data increase from 6 bits per symbol up to 8 bits per symbol, a 33% increase. But the 6dB increase in power could alternately allow a 4X expansion of the bandwidth, resulting in a 400% increase in channel capacity!

Another case that can be examined is what happens to data capacity when a second (or a third) user with similar attenuation must be accommodated in the same wide band with equal priority. Two cases can be evaluated, upstream or downstream.

Upstream, assume the power limitation is set in the CM's final amplifier. If both users transmit sequentially half the time, each with maximum bandwidth, they both get half of the total data capacity. If they both transmit at the same time but at different frequencies (e.g., using OFDMA), sharing the bandwidth they both get better performance because a 3dB better signal to noise is available due to the bandwidth reduction. This could potentially allow going to a higher order modulation.

Downstream, it is a wash because a maximum transmit power is fixed. You get the same capacity if you transmit for twice as long in half the bandwidth.

Looking a little deeper at Shannon's reference paper [1], a question is answered about how to best utilize available power density to maximize data throughput in a wide channel where the signal to noise varies with frequency. Because of increases in cable attenuation between 1 GHz and 1.7 GHz, a variation in S/N would be expected if transmit power were flat vs. frequency. Shannon’s answer (his equation 34 [1]) is that the signal density at any frequency plus the noise density at that frequency should be constant for maximum capacity. For frequencies where the noise power is low, the signal power should be high, and vice versa. This is not how cable currently operates in the downstream, where amplifiers launch signals with an up-tilt to compensate for loss at higher frequencies. Currently the design for the cable downstream path is optimized for analog TV signal delivery, not maximum data throughput.

Appendix A analyzes Shannon's limits for two cases, flat transmit power at all frequencies (Table 1), and constant signal to noise ratio at all frequencies (Table 2).

3 CONCLUSION

If possible, use wide bandwidth transmissions above 1 GHz for best performance when transmit power is limited.

APPENDIX A ANALYSIS OF A CABLE PLANT FOR RANDOM NOISE

Assume a downstream amplifier has an available transmit power of +65 dBmV and it is transmitting into an available bandwidth of 700 MHz between 1000 and 1700 MHz. (This analysis applies equally well for upstream or downstream.) All of the signal power is being directed to one user's receiver. The noise figure of the receiver is 9dB, but the biggest problem is the attenuation of the coaxial cable at high frequencies. Assume the loss at 1000 MHz of the half-inch hard line is 2.53dB per 100 feet and the loss of the RG-6 drop cable is 7.87 per 100 feet. Also assume 800 feet of hard line and 200 feet of drop cable are used. Allow a 7 dB flat insertion loss for 4 taps in cascade. Results of spreadsheet analysis are shown in Table 1 and in Table 2.

Note that the flat transmit power spectrum case is slightly better as predicted, but not dramatically so. This is because a flat transmit spectrum delivers a better signal to noise at lower frequencies, where attenuation is lower.

Table 1. Uniform Transmit Power Density

Frequency MHz	Capacity Mb
1000-1050	808.5234162
1050-1100	790.2118082
1100-1150	772.1425466
1150-1200	754.3000111
1200-1250	736.6701998
1250-1300	719.2405062
1300-1350	701.9995352
1350-1400	684.9369499
1400-1450	668.0433437
1450-1500	651.3101321
1500-1550	634.7294626
1550-1600	618.2941377
1600-1650	601.9975504
1650-1700	585.8336299
Shannon Capacity Mb=	9728.233229

Table 2. Constant Receive S/N

Frequency MHz	Capacity Mb
1000-1050	664.392832
1050-1100	664.392832
1100-1150	664.392832
1150-1200	664.392832
1200-1250	664.392832
1250-1300	664.392832
1300-1350	664.392832
1350-1400	664.392832
1400-1450	664.392832
1450-1500	664.392832
1500-1550	664.392832
1550-1600	664.392832
1600-1650	664.392832
1650-1700	664.392832
Shannon Capacity Mb=	9301.49965

APPENDIX B REFERENCES

1. "Communications in the Presence of Noise" by Claude E. Shannon, Proceedings of the IEEE, VOL 86, No2, Feb. 1998, http://www.stanford.edu/class/ee104/shannonpaper.pdf